Dark energy from the gas of wormholes

A. A. Kirillov
Izhorskaya Str. 27-22, Nizhny Novgorod, 603105, Russia
E.P. Savelova
Bulvar Festivalny, 17-67, Uljanovsk, 432067, Russia

Abstract

The observed dark energy phenomenon is attributed to the presence of the zero-point fluctuations of matter fields. We show that due to the presence of the gas of virtual wormholes the zero-point energy is finite and forms the finite (of the Planckian order) value. The observed value of the cosmological constant is somewhat reduced due to the two effects. First is the renormalization of the gravitational constant which presumably forms the initial (local) value H_{inf}^2 predicted by the inflationary scenario. And second is an additional reduction due to the presence of the gas of actual wormholes required by the dark matter phenomenon. We show that the Starobinsky model of inflation explains both, the inflationary stage in the past and the present day acceleration of the Universe. It also represents the so-called model of the eternal Universe.

1 Introduction

As is well known modern astrophysics (and, even more generally, theoretical physics) faces two key problems. Those are the nature of dark matter and dark energy. Recall that more than 90% of matter of the Universe has a non-baryonic dark (to say, mysterious) form, while lab experiments still show no evidence for the existence of such matter. Both dark components are intrinsically incorporated in the most successful ΛCDM (Lambda cold dark matter) model which reproduces correctly properties of the Universe at very large scales (e.g., see [1] and references therein). The only failure of ΛCDM, i.e., the presence of cusps ($\rho_{DM} \sim 1/r$) in centers of galaxies [2], can be cured if, instead of non-baryon particles, we shall use wormholes (see for detail [3]). Indeed, wormholes represent extremely heavy (in comparision to particles) objects which at very large scales behave exactly like non-baryon cold particles, while at smaller scales (in galaxies) they strongly interact with baryons and form the observed [4] cored ($\rho_{DM} \sim const$) distribution. It worth expecting that wormholes will play the central role in the explanation of the dark matter phenomenon.

Save the dark matter component ΛCDM requires the presence ($\sim 70\%$) of dark energy (of the cosmological constant). Moreover, there are strong evidences

for the start of the acceleration phase in the evolution of the Universe [5]. In the present paper we shall try to demonstrate that wormholes play here the central role as well. It turns out that virtual wormholes are responsible for the formation of the renormalized (finite) value of the cosmological constant, while the observed value is somewhat reduced by the presence of the gas of actual wormholes.

It seems that the observed small acceleration of the Universe is the start of a sequent inflationary stage predicted by the Starobinsky model [6]. Recall that the first inflationary model suggested in Ref. [6] is based on one-loop vacuum corrections to the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \gamma \left\langle T_{\mu\nu} \right\rangle \tag{1}$$

where the expectation value $\langle T_{\mu\nu} \rangle$ describes the energy of zero-point fluctuations of matter fields. In the absence of particles (vacuum state) $\gamma \langle T_{\mu\nu} \rangle \sim \Lambda = const$ which immediately launches the inflationary phase. Such a phase was shown to be unstable in both directions. During the expansion the inflation ends due to the particle production, while in the backward direction the presence of an arbitrary small amount of matter (particles) destroys the inflationary phase as well (for on the last stage of the collapse the ordinary matter always prevails the cosmological constant).

Wormholes add to the above scenario only two features. First, virtual wormholes remove divergencies and make the energy of zero-pint fluctuations be finite. This, in principle, allows to account for higher order corrections to the above equation. And the second feature is that wormholes always remove some portion of degrees of freedom and, therefore, they can be described by ghost fields (by the analogy with Fadeev-Popov ghosts). The inflationary phase is always accompanied with the production of wormholes (or ghost particles) as well, which leads to an additional reduction (i.e., a partial decay) of the cosmological constant (zero-point energy). On the subsequent stage wormholes created can merge reheating thus the rest matter. This gives the standard Friedman stage of the evolution which lasts until the cosmological constant prevails again the matter (as we do observe now) and all the picture repeats. It is impossible to say on which cycle of this eternal process we are. Thus the Starobinsky model seems to give the simplest model of the eternal Universe.

Since the divergencies in QFT (quantum field theory) come from the ultraviolet behavior which involves extremely small scales, we may expect that effects of general relativity are not essential here and in the present paper we merely neglect them to the leading order. We also use everywhere the Planckian units. In the present paper we also use only mass-less wormholes [8, 3]. In general massive wormholes may also appear but we expect that they are essentially suppressed (from the energetic standpoint). Moreover, we may expect that basic topological effects will not change. We point out that wormhole rest masses do appear in the presence of particles created as it was explained in [3] which leads to some additional non-linear phenomena on the inflationary stage. This however requires an independent investigation.

The paper is organized as follows. In Sec. 2 we present the construction of the generating functional with virtual wormholes taken into account. In Sec. 3 we investigate properties of the two-point Green function. We show that the presence of the gas of virtual wormholes can be described by the topological bias [7] exactly as it happens in the presence of actual wormholes [8, 3]. We demonstrate that the mean value for the bias defines the cutoff function in the space of modes. In Sec. 4 we explicitly demonstrate that for a particular set of wormholes the bias defines not more than the projection operator on the subspace of functions obeying to the proper boundary conditions at wormhole throats. The projective nature of the bias means that wormholes merely cut some portion of degrees of freedom (modes). Phenomenologically it means that wormholes can be described by the presence of ghost fields which compensate the extra (cut by wormholes) modes. In Sec. 5 we show how the cutoff expresses via some dynamic parameters of wormholes. The exact definition of such parameters we leave for the future investigation. In Sec. 6 we demonstrate that wormholes lead to a finite (of the planckian order) value of $\langle T_{\mu\nu}\rangle$ which requires considering the contribution from the smaller and smaller wormholes with divergent density $n \to \infty$. In Sec. 7 we define the renormalization of the gravitational constant which leads to a reduction of the observed value of Λ. Recall that inflationary scenarios require $\sqrt{\Lambda/3} = H_{inf} \sim 10^{-5} \sim \Delta T/T$ which in our approach means that $\gamma \sim 10^{-10}\gamma_0$, where γ_0 is the naked gravitational constant. In Sec. 8 we consider actual wormholes and show that they produce a negative contribution to the cosmological constant. Thus, the creation of wormholes during the inflationary stage leads to a partial decay of the initial cosmological constant. In Sec.9 we repeat basic results an discuss some perspectives.

2 Generating function

The basic aim of this section is to construct the generating functional which can be used to get all possible correlation functions. Consider the partition function which includes the sum over topologies and the sum over field configurations

$$Z_{total} = \sum_{\tau} \sum_{\varphi} e^{-S}. \tag{2}$$

For the sake of simplicity we shall use from the very beginning the Euclidean approach (e.g., see Refs. [9]-[12] and references therein), i.e., the action in the form

$$S = -\frac{1}{2}\left(\varphi \hat{A} \varphi\right) + (J\varphi) \tag{3}$$

and the notions $(J\varphi) = \int J(x)\,\varphi(x)\,d^4x$, where the integral is taken over physically admissible portion of R^4 only. From the variation principle $\delta S = 0$ we may find the equation of motions for the field φ

$$-\hat{A}\varphi + J = 0 \tag{4}$$

which has the solution ($\hat{A}^{-1}$ is the Green function obeying to a proper boundary conditions)

$$\varphi = \hat{A}^{-1} J. \tag{5}$$

2.1 Sum over fields

We fix the topology of space by placing a set of wormholes with parameters ξ_i (i.e., $\xi_i = \left(a_i, R_i^+, R_i^-\right)$, where a is the throat radius and $R^\pm$ are positions of throats in space). For general properties of a wormhole see Ref.[3]. Then we consider the sum over field configurations φ, which can be replaced by the integral

$$Z^*\left(J\right) = \int \left[D\varphi\right] e^{\frac{1}{2}\left(\varphi \hat{A} \varphi\right) - \left(J\varphi\right)}. \tag{6}$$

Upon the simple transformations

$$\frac{1}{2}\left(\varphi \hat{A} \varphi\right) - \left(J\varphi\right) = \frac{1}{2}\left(\widetilde{\varphi} \hat{A} \widetilde{\varphi}\right) - \frac{1}{2}\left(J \hat{A}^{-1} J\right), \tag{7}$$

where $\widetilde{\varphi} = \varphi - \hat{A}^{-1} J$, we cast the partition function to the form

$$Z^* = \int \left[D\widetilde{\varphi}\right] e^{\frac{1}{2}\left(\widetilde{\varphi} \hat{A} \widetilde{\varphi}\right) - \frac{1}{2}\left(J \hat{A}^{-1} J\right)} = Z_0(\hat{A}) e^{-\frac{1}{2}\left(J \hat{A}^{-1} J\right)}, \tag{8}$$

where $Z_0(\hat{A}) = \int \left[D\varphi\right] e^{\frac{1}{2}\left(\varphi \hat{A} \varphi\right)}$ is the standard expression and $\hat{A}^{-1} = A^{-1}\left(\xi_1, ..., \xi_N\right)$ is the Green function for a fixed topology, i.e., for a fixed set of wormholes $\xi_1, ..., \xi_N$.

In the case of a fixed topology the generating functional (6)-(8) allows us to construct the perturbation scheme, when we add to the action (3) the interaction term $\Delta S\left(\varphi\right)$, by means of using the obvious expression

$$Z\left(J\right) = e^{-\Delta S\left(-\frac{\delta}{\delta J}\right)} Z^*\left(J\right) = \sum_{n=0}^{\infty} \frac{\left(-1\right)^n}{n!} \left(\Delta S\left(-\frac{\delta}{\delta J}\right)\right)^n Z^*\left(J\right) \tag{9}$$

and generate all possible momenta (the higher order Green functions $G_s\left(x_1, x_2, ...x_s\right)$) as

$$\left\langle \varphi\left(x_1\right) ... \varphi\left(x_s\right) \right\rangle = \frac{1}{Z\left(J\right)} \left(-\frac{\delta}{\delta J(x_1)}\right) ... \left(-\frac{\delta}{\delta J(x_s)}\right) Z\left(J\right)\bigg|_{J=0} \tag{10}$$

which depend on parameters of wormholes, i.e., $G_s = G_s\left(\xi_1, ..., \xi_N\right)$.

2.2 Sum over wormholes

Consider now the sum over topologies τ. To this end we restrict with the sum over the number of wormholes and integrals over parameters of wormholes:

$$\sum_{\tau} \rightarrow \sum_{N} \int \prod_{i=1}^{N} d\xi_i \tag{11}$$

where in general the integration over parameters is not free (e.g., it obeys the obvious restriction $\left|\vec{R}_i^+ - \vec{R}_i^-\right| \geq 2a_i$). This defines the generating function as

$$Z_{total}\left(J\right) = \sum_N \int \left(\prod_{i=1}^N d\xi_i\right) Z_0(\hat{A}) e^{-\frac{1}{2}\left(J\hat{A}^{-1}J\right)}. \tag{12}$$

The sum over topologies assumes an additional averaging out for the Green functions (10) with the measure $d\mu_N = \rho\left(\xi, N\right) d^N\xi$, where

$$\rho\left(\xi, N\right) = \frac{Z_0(\hat{A}\left(\xi, N\right))}{Z_{total}\left(0\right)} \tag{13}$$

which obey the obvious normalization condition $\sum_N \int d\mu_N = \sum_N \rho_N = 1$. The averaging out over topologies assumes the two stages. First we fix the total number of wormholes N and average over the parameters of wormholes ξ (i.e., over parameters of a static gas of wormholes in R^4). Then we sum over the number of wormholes N (the so-called big canonical ensemble).

The basic difficulty of the standard field theory is that the perturbation scheme based on the decomposition (9) leads to divergent expressions. This remains true for every particular topology of space (i.e., for any particular set of wormholes), since there always exists a scale below which the space looks like the ordinary Euclidean space. What we expect is that the sum over all possible topologies will remove such a difficulty.

3 The two-point Green function

From (8), (9), and (10) we see that the very basic role in QFT plays the two point Green function. Such a Green function can be found from the equation

$$\hat{A}G(x, x') = -\delta(x - x') \tag{14}$$

with proper boundary conditions at wormholes, which gives $G = A^{-1}$. Now let us introduce the bias function $N(x, x')$ as

$$G(x, y) = \int G_0(x, x')N(x', y)dx', \tag{15}$$

where $G_0(x, x')$ is the ballistic (or the standard Euclidean Green function) and the bias can be presented as

$$N(x, x') = \delta(x - x') + \sum_i b_i\delta(x - x_i) \tag{16}$$

where b_i are fictitious sources at positions x_i which should be added to obey the proper boundary conditions.

In the simplest approximation (of a rarefied gas) the bias function can be expressed via the bias for a single wormhole. Indeed if we use the decomposition $N(x,x') = \delta(x,x') + b\,(x,x')$, then

$$b(x,x') = N \int b_1(x,x',\xi) F(\xi) d\xi \tag{17}$$

where $b_1(x,x',\xi)$ is the bias function b for a single wormhole, N is the total number of wormholes, while

$$F\,(\xi) = \frac{1}{N} \sum_{i=1}^{N} \delta\,(\xi - \xi_i) \tag{18}$$

is the density of wormholes in the configuration space ξ. In what follows we shall use the notion

$$n_1\,(x,x',\xi) = \delta(x-x') + N b_1\,(x,x',\xi)$$

which, in virtue of $\int F(\xi)d\xi = 1$, defines the total bias function as

$$N(x,x') = \int n_1(x,x',\xi) F(\xi) d\xi. \tag{19}$$

We point out that the superposition of wormholes in (19) represents only an approximation (which works when we retain only the first order images by the parameter $\frac{a}{|R^- - R^+|} \ll 1$ (e.g., see Ref. [8]), while in the most general case the superposition does not work and $N(x,x') = N(x,x',\xi_1,...\xi_N)$).

Now taking into account that due to (19) (15) the total Green function can be expressed via the auxiliary Green function $\hat{G}_1\,(x,y,\xi) = \int G_0(x,x')n_1(x',y)dx'$ (which in the case $N=1$ gives the Green function for a single wormhole) as $G = \int \hat{G}_1\,(\xi)\,F(\xi)d\xi$. Let us introduce the notion

$$\left\{J\hat{G}J\right\} = \int \left(J\hat{G}_1 J\right) F(\xi) d\xi, \tag{20}$$

then for the sum over wormholes (11) we may replace

$$Z_{total}\,(J) = \int [DF]\, Z_0(\hat{G}^{-1}) e^{-\frac{1}{2}\left\{J\hat{G}J\right\}} \tag{21}$$

where $\int [DF] = \sum\limits_N \int \prod\limits_{i=1}^{N} d\xi_i$.

The action (3) remains invariant under translations $\vec{x}' = \vec{x} + \vec{c}$ with an arbitrary $\vec{c}$ which means that the measure (13) does not actually depend on the position of the center of mass of the gas of wormholes and, therefore, we may restrict ourself with homogeneous distributions $F(\xi)$ of wormholes in space only. Indeed, we may define $d^N\xi = d^N\xi' d^4c$, while the integration over d^4c gives the

volume of R^4 i.e., $\int d^4c = L^4 = V$ which disappears from (13) due to the denominator[1]. In what follows we shall omit the prime from ξ'.

Let us consider the Fourier representation $N(x,x',\xi) \to N(k,k',\xi)$ which in the case of a homogeneous distribution of wormholes gives $N(k,k') = N(k,\xi)\delta(k-k')$, then we find

$$G(k) = G_0(k)\, N(k,\xi)$$

and the Green function can be taken as

$$G = \frac{N(k,\xi)}{k^2+m^2}. \tag{22}$$

Then for the total partition function we find

$$Z_{total}(J) = \int [DN(k)]\, e^{-\sigma(N(k))} Z_0(N(k)) e^{-\frac{1}{2}\frac{L^4}{(2\pi)^4}\int\left(\frac{N(k)}{k^2+m^2}|J_k|^2\right)dk}, \tag{23}$$

where $[DN] = \prod\limits_k dN_k$ and $\sigma(N)$ comes from the integration measure (i.e., from the Jacobian of transformation from $F(\xi)$ to $N(k)$)

$$e^{-\sigma(N)} = \int [DF]\, \delta\left(N(k) - N(k,\xi)\right).$$

We point out that $\sigma(N)$ can be changed by means of adding to the action (3) of an arbitrary "non-dynamical" constant term which depends only on topology (wormholes) $S \to S + \Delta S(N(k))$. The multiplier $Z_0(N)$ defines the simplest measure for topologies which is given by

$$Z_0 = \exp\left(-\int \Lambda d^4x\right) = \exp\left\{\pm\frac{1}{2}\frac{L^4}{(2\pi)^4}\int \ln\left(\frac{\pi}{k^2+m^2}\right) N(k) d^4k\right\}, \tag{24}$$

where $\pm$ stands for Bose/Fermi statistics of the field φ. Now by means of using the expression (23) and (10) we find the two-point Green function in the form

$$G(k) = \frac{\overline{N}(k)}{k^2+m^2} \tag{25}$$

where $\overline{N}(k)$ is the cutoff function (the mean bias) which is given by

$$\overline{N}(k) = \sum_N \int d\mu_N N(k,\xi) = \frac{1}{Z_{total}(0)} \int [DN]\, e^{-\sigma(N)} Z_0(N)\, N(k).$$

At the present stage we still cannot evaluate the exact form for the cutoff function $\overline{N}(k)$ in virtue of the ambiguity of $\Delta S(N(k))$ pointed out. Such a term may include two parts. First part $\Delta_1 S$ describes the proper dynamics of wormholes and should be considered separately, while the second part may describe

[1] Technically, we may first restrict a portion of R^4 in the (3) to a finite volume V and then in final expressions consider the limit $V \to \infty$ (which represents the standard tool in thermodynamics and QFT).

"external conditions" for the intensity of topology fluctuations. Actually the last term can be used to prescribe an arbitrary particular value for the cutoff function $\overline{N}(k) = f(k)$. Indeed, the "external conditions" can be accounted for by adding the term $\Delta_2 S = (\lambda, N) = \int \lambda(k) N(k) d^4k$, where $\lambda(k)$ plays the role of a specific chemical potential which implicitly depends on $f(k)$ through the equation

$$f(k) = \frac{1}{Z_{total}(\lambda, 0)} \int [DN] e^{(\lambda,N)-\sigma(N)} N(k),$$

where the value of Z_0 (24) is included into the value of the chemical potential $\lambda(k)$. From the QFT standpoint such a term leads merely to a renormalization of the cosmological constant. By other words the intensity of topology fluctuations (i.e., the cutoff function) is driven by the cosmological constant Λ and vise versa.

4 Topological bias as a projection operator

By the construction the topological bias $N(x, x')$ plays the role of a projection operator onto the space of functions (a subspace of functions on R^4) which obey the proper boundary conditions at throats of wormholes. This means that for any particular topology (for a set of wormholes) there exists the basis $\{f_i(x)\}$ in which it takes the diagonal form $N(x, x') = \sum N_i f_i(x) f_i^*(x')$ with eigenvalues $N_i = 0, 1$ (since $N_i^2 = N_i$). In this section we illustrate this simple fact (which is probably not obvious for readers) by the explicit construction of the reference system for a single wormhole when physical functions become (due to the boundary conditions) periodic functions of one of coordinates.

Indeed, consider a single wormhole with parameters ξ (i.e., $\xi = (a, R^+, R^-)$, where a is the throat radius and $R^\pm$ are positions of throats in space[2]). Consider now a particular solution ϕ_0 to the equation $\Delta\phi_0 = 0$ (harmonic function) for R^4 in the presence of the wormhole[3], which corresponds to the situation when throats possess a unit charge/mass but those have the opposite signs. Now define the family of lines of force $x(s, x_0)$ which obey the equation $\frac{dx}{ds} = -\nabla\phi_0(x)$ with initial conditions $x(0) = x_0$. Physically, such lines correspond to lines of force for a two charged particles in positions $R^\pm$ with charges ± 1. We note that all points which lay on the trajectory $x(s, x_0)$ may be taken as initial conditions and they define the same line of force with the obvious redefinition $s \to s - s_0$. By other words we may take as a new coordinates the parameter s and portion of the coordinates orthogonal to the family of lines $x_0^\perp$. Coordinates $x_0^\perp$ can be taken

[2]In general, there exists an additional parameter U_β^α which defines a rotation of one of throats before gluing. However, it does not change the subsequent construction. There always exists a diffeomorfic map of coordinates $x' = h(x)$ which sets such a matrix to unity.

[3]Instead of the construction used here one may use also another method. Indeed, consider two point charges, then the function $\phi_0 = 1/(x - x_+)^2 - 1/(x - x_-)^2$ can be taken as a new coordinate. Wormhole appears when we identify (glue) surfaces $\phi_0 = \pm\omega$. We point out that such surfaces are not spheres, though they reduce to spheres in the limit $|x_+ - x_-| \to \infty$ or $\omega \to \infty$.

as laying in the hyperplane R^3 which is orthogonal to the vector $\vec{d} = \vec{R}^- - \vec{R}^+$ and goes through the point $\vec{X}_0 = (\vec{R}^- + \vec{R}^+)/2$. Let $s^\pm\left(x_0^\perp\right)$ be the values of the parameter s at which the line intersects the throats $R^\pm$. Then instead of s we may consider a new parameter θ as $s\left(\theta\right) = s^- + \left(s^+ - s^-\right)\theta/2\pi$, so that when $\theta = 0, 2\pi$ the parameter s takes the values $s = s^-, s^+$ respectively. The gluing procedure at throats means merely that we identify points at $\theta = 0$ and $\theta = 2\pi$ and all physical functions in the space R^4 with a single wormhole ξ become periodic functions of θ. Thus, the coordinate transformation $x = x(\theta, x_0^\perp)$ gives the map of the above space onto the cylinder with a specific metric $dl^2 = \left(d\vec{x}\left(\theta, x_0^\perp\right)\right)^2 = g_{\alpha\beta}dy^\alpha dy^\beta$ (where $y = (\theta, x_0^\perp)$) whose components are also periodic in terms of θ. Now we can continue the coordinates to the whole space R^4 (to construct a cover of the fundamental region $\theta \in [0, 2\pi]$) simply admitting all values $-\infty < \theta < +\infty$ this, however, requires to introduce the bias

$$\frac{1}{\sqrt{g}}\delta\left(\theta - \theta'\right) \to N\left(\theta - \theta'\right) = \sum_{n=-\infty}^{+\infty} \frac{1}{\sqrt{g}}\delta\left(\theta - \theta' + 2\pi n\right),$$

since every point and every source in the fundamental region acquires a countable set of images in the non-physical region (inside of wormhole throats). Considering now the Fourier transforms for θ we find

$$N\left(k, k'\right) = \sum_{n=-\infty}^{+\infty} \delta\left(k - n\right)\delta\left(k - k'\right).$$

We point out that the above bias gives the unit operator in the space of periodic functions of θ. From the standpoint of all possible functions on R^4 it represents the projection operator $\widehat{N}^2 = \widehat{N}\left(\xi\right)$ (taking an arbitrary function f we find that upon the projection $f_N = \widehat{N}f$ f_N becomes a periodic function of θ, i.e.,only periodic functions survive).

The above construction can be easily generalized to the presence of a set of wormholes. In the approximation of a dilute gas of wormholes we may neglect the influence of wormholes on each other (at least there always exists a sufficiently smooth map which transforms the family of lines of force for "independent" wormholes onto the actual lines). Then the total bias (projection) may be considered as the product

$$N_{total}\left(x, x'\right) = \int \left(\prod_i \sqrt{g_i} d^4 y_i\right) N\left(\xi_1, x, y_1\right) N\left(\xi_2, y_1, y_2\right) ... N\left(\xi_N, y_{N-1}, x'\right),$$

where $N\left(\xi_i, x, x'\right)$ is the bias for a single wormhole with parameters ξ_i. Every such a particular bias $N\left(\xi_i, x, x'\right)$ realizes projection on a subspace of functions which are periodic with respect to a particular coordinate $\theta_i\left(x\right)$, while the total bias gives the projection onto the intersection of such particular subspaces (functions which are periodic with respect to every parameter θ_i).

5 Cutoff

The projective nature of the bias operator $N(x,x')$ allows us to express the cutoff function $\overline{N}(k)$ via dynamical parameters of wormholes. Indeed, consider a box L^4 in R^4 and periodic boundary conditions which gives $k = 2\pi n/L$ (in final expressions we consider the limit $L \to \infty$, which gives $\sum_k \to \frac{L^4}{(2\pi)^4}\int d^4k$). And let us consider the decomposition for the integration measure in (23) as

$$\sigma(N) = \sigma_0 + \sum \lambda_1(k)\, N(k) + \frac{1}{2}\sum \lambda_2(k,k')\, N(k)\, N(k') + ...$$

where $\lambda_1(k)$ includes also the contribution from $Z_0(k)$. We point out that this measure plays the role of the action for the bias $N(k)$. Indeed, the variation of the above expression gives the equation of motions for the bias in the form

$$\sum_{k'} \lambda_2(k,k')\, N(k') = -\lambda_1(k)$$

which can be found by considering the proper dynamics of wormholes. We however do not consider the problem of the dynamical description of wormholes here and leave this for the future research. Then in the first approximation we may retain the linear term only. Then taken into account that $N(k) = 0, 1$ $(N^2 = N)$ we find

$$\overline{N}(k) = \frac{1}{Z_{total}(k)} \sum_{N=0,1} e^{-\lambda_1(k)N(k)} N(k) = \frac{e^{-\lambda_1(k)}}{1 + e^{-\lambda_1(k)}}. \tag{26}$$

The simplest choice gives merely $\lambda_1(k) = -\sum \ln Z_0(k)$, where the sum is taken over the number of fields and $Z_0(k)$ is given by $Z_0(k) = \sqrt{\pi/(k^2+m^2)}$. In the case of a set of massless fields we find $\overline{N}(k) = Z(k)/(1+Z(k))$ where $Z(k) = (\sqrt{\pi}/k)^{\alpha}$ and α is the number of degrees of freedom. To ensure the absence of divergencies one has to consider the number of fields $\alpha > 4$ [7]. However, as we show in the next section such a choice cannot be correct. Indeed, while its behavior at very small scales (i.e., when exceeding the Plankian scales $Z(k) \lesssim 1$ and $\overline{N}(k) = Z(k)$) may be physically accepted, since it produces some kind of a cutoff, on the mass-shell $k^2 + m^2 \to 0$ it gives the behavior $\overline{N}(k) \to 1$ which is merely incorrect (the true behavior is $\overline{N}(k) \to const < 1$).

6 Cosmological constant

In this section we consider the renormalization of the stress energy tensor. Let N be the fixed number of wormholes. Then the stress energy tensor can be obtained directly from the two-point green function (22), (25) as

$$-\langle T_{\alpha\beta}(x)\rangle = \lim_{x'\to x}\left(\partial_\alpha \partial'_\beta - \frac{1}{2} g_{\alpha\beta}\left(\partial^\mu \partial'_\mu + m^2\right)\right)\langle G(x,x',\xi)\rangle . \tag{27}$$

By means of using the Fourier transform $G(x,x',\xi) = \int e^{-ik(x-x')} G(k,\xi) \frac{d^4k}{(2\pi)^4}$ and the expression (22), (25) we arrive at

$$-\langle T_{\alpha\beta}(x)\rangle = -\frac{1}{4} g_{\alpha\beta} \int \left(1 + \frac{m^2}{k^2+m^2}\right) \langle N(k,\xi)\rangle \frac{d^4k}{(2\pi)^4}, \tag{28}$$

where the property $\int k_\alpha k_\beta f(k)\, d^4k = \frac{1}{4} g_{\alpha\beta} \int k^2 f(k)\, d^4k$ has been used.

For the sake of simplicity we consider the massless case. Then by the use of the cutoff $\overline{N}(k) = \pi^{\alpha/2} / \left(\pi^{\alpha/2} + k^\alpha\right)$ from the previous section we get the finite value ($\alpha > 4$ is the number of the field helicity states)

$$\Lambda = \frac{1}{4}\alpha \int \frac{\pi^{\alpha/2}}{\pi^{\alpha/2}+k^\alpha} \frac{d^4k}{(2\pi)^4} = \frac{1}{32}\Gamma\left(\frac{\alpha-4}{\alpha}\right)\Gamma\left(\frac{4}{\alpha}\right). \tag{29}$$

Since the leading contribution comes here from very small scales, we may hope that this value will not essentially change if the true cutoff function changes the behavior on the mass-shell as $k \to 0$ (e.g., if we take $\lambda_1(k) = -\sum \ln Z_0(k) + \delta\lambda(k)$ with $\delta\lambda(k) \ll \ln Z_0(k)$ as $k \gg 1$).

To understand how wormholes remove divergencies, it will be convenient to split the bias function into two parts $N(k,\xi) = 1 + b(k,\xi)$, where 1 corresponds to the standard Euclidean contribution, while $b(k,\xi)$ is the contribution of wormholes. The first part gives the well-known divergent contribution of vacuum field fluctuations $\left\langle T^0_{\alpha\beta}\right\rangle = \Lambda g_{\alpha\beta}$ with $\Lambda \to +\infty$, while the second part remains finite for any finite number of wormholes and, due to the projective nature of the bias described in the previous section, it partially compensates (reduces) the value of the cosmological constant, i.e., $\langle \Delta T_{\alpha\beta}\rangle = -\delta\Lambda g_{\alpha\beta}$, where $\delta\Lambda = \sum_N \rho_N \delta\Lambda(N)$ and $\delta\Lambda(N)$ is a finite contribution of a finite set of wormholes.

6.1 The long-wave behavior of the bias (rarefied gas approximation)

To illustrate the compensational role of virtual wormholes we consider now the bias for a particular set of wormholes. For the sake of simplicity we consider the case when $m = 0$. Consider the Green function for the Laplace equation

$$-\Delta G(x,x') = \delta(x-x')$$

in the presence of a single wormhole. A single wormhole can be viewed as a couple of conjugated spheres $S^3_\pm$ of the radius a with a distance $d = \left|\vec{R}_+ - \vec{R}_-\right|$ between centers of spheres. So the parameters of the wormhole are[4] $\xi = (a, R_+, R_-)$. The interior of the spheres is removed and surfaces are glued together. The Green function for the Euclidean space is merely $G_0(x,x') =$

[4] We do not account here for the additional parameter (rotation matrix U), since in the approximation used such parameter gives not essential contribution.

$\frac{1}{4\pi^2}\frac{1}{(x-x')^2}$ $(G_0(k) = 1/k^2)$. The actual Green function can be found by means of using the image method (the straightforward generalization of results in Ref. [8]). Then the proper boundary conditions (the actual topology) can be accounted for by adding the bias of the source

$$\delta(x-x') \to \delta(x-x') \ + b(x,x')\,. \tag{30}$$

In the approximation $a/d \ll 1$ (e.g., see for details Ref.[8]) the bias takes the form

$$b_1(x,x',\xi) = a^2\left(\frac{1}{(R_- - x')^2} - \frac{1}{(R_+ - x')^2}\right)\left[\delta(\vec{x}-\vec{R}_+) - \delta(\vec{x}-\vec{R}_-)\right]. \tag{31}$$

This form for the bias is convenient when constructing the true Green function and considering the long-wave limit, however it is not acceptable in considering the short-wave behavior and vacuum polarization effects. Indeed, the positions of additional sources are in the physically non-admissible region of space (the interior of spheres $S^3_\pm$). To account for the finite value of the throat size we should replace in (31) the point-like source with the surface density (induced on the throat) i.e.,

$$\delta(\vec{x}-\vec{R}_\pm) \to \frac{1}{2\pi^2 a^3}\delta(\left|\vec{x}-\vec{R}_\pm\right| - a). \tag{32}$$

Such a replacement does not change the value of the true Green function, however now all extra sources are in the physically admissible region of space.

We see that the bias function for the gas of wormholes in this approximation is additive, i.e.,

$$b_{total}(x,x') = \sum b_1(x,x',\xi_i) = N\int b_1(x,x',\xi)F(\xi)d\xi. \tag{33}$$

For a homogeneous and isotropic distribution $F(\xi) = F(a,d)$, then for the bias we find

$$b_{total}(x-x') = \int \frac{1}{\pi^2 a}\left(\frac{1}{R_-^2} - \frac{1}{R_+^2}\right)\delta(\left|\vec{x}-\vec{x}'-\vec{R}_+\right| - a)NF(a,|R_+ - R_-|)\,d\xi \tag{34}$$

Consider the Fourier transform $F(a,X) = \int F(a,k)\,e^{-ikX}\frac{d^4k}{(2\pi)^4}$ and using the integral $\frac{1}{x^2} = \int \frac{4\pi^2}{k^2}e^{-ikx}\frac{d^4k}{(2\pi)^4}$ we find for $b(k) = \int b(x)\,e^{ikx}d^4x$ the expression

$$b_{total}(k) = 2N\int a^2\frac{4\pi^2}{k^2}\left(F(a,k) - F(a,0)\right)\frac{J_1(ka)}{ka/2}da. \tag{35}$$

Consider now a particular form for $F(a,X)$, e.g.,

$$NF(a,X) = \frac{n}{2\pi^2 r_0^3}\delta(a-a_0)\,\delta(X-r_0)\,, \tag{36}$$

where $n = N/V$ is the density of wormholes. In the case $N = 1$ this function corresponds to a single wormhole with the throat size a_0 and the distance

between throats $r_0 = |R_+ - R_-|$. We recall that the action (3) remains invariant under translations and rotations which straightforwardly leads to the above function. Then $NF(a,k) = \int NF(a,X)\, e^{ikx} d^4x$ reduces to $NF(a,k) = n\frac{J_1(kr_0)}{kr_0/2}\delta(a-a_0)$. Thus from (35) we find

$$b(k) = -2na^2\frac{4\pi^2}{k^2}\left(1 - \frac{J_1(kr_0)}{kr_0/2}\right)\frac{J_1(ka)}{ka/2}. \tag{37}$$

As $k \to 0$ we get $J_1(kr_0)/\frac{kr_0}{2} \approx 1 - \frac{1}{2}\left(\frac{kr_0}{2}\right)^2 + ...$ which gives $b(k) \approx -\pi^2 na^2 r_0^2 + ...$ In a more general case we find that on the mass-shell ($k \to 0$) $b(k) \approx -\int \pi^2 a^2 r_0^2 n(a, r_0)\, da dr_0 + ...$, where $n(a, r_0)$ is the density of wormholes with a particular values of a and r_0, and for the cutoff function (26) we get

$$\overline{N}(k) \to 1 - \int \pi^2 a^4 n(a, r_0)\frac{r_0^2}{a^2} da dr_0 < 1. \tag{38}$$

Thus, we see that on the mass-shell the presence of virtual wormholes diminishes the value of the cutoff function, which should lead to a specific renormalization of the field, rest mass, and charge values.

6.2 Renormalization of the cosmological constant

Consider now the contribution of virtual wormholes to the cosmological constant which is given by $\delta\Lambda(N) = -\frac{1}{4}\int b(k)\frac{d^4k}{(2\pi)^4} = -\frac{1}{4}b_{total}(0)$. Then from the expressions (34) and (36) we get

$$b_{total}(0) = -\frac{n}{2\pi^4 a^3 r_0^3}\int\left(1 - \frac{a^2}{R_-^2}\right)\delta(R_+ - a)\delta\left(|R_+ - R_-| - r_0\right) d^4R_- d^4R_+$$

which gives

$$b_{total}(0) = -2n\left(1 - f\left(\frac{a}{r_0}\right)\right),$$

where

$$f\left(\frac{a}{r_0}\right) = \frac{2}{\pi}\int_0^\pi \frac{a^2\sin^2\theta d\theta}{a^2 + 2ar_0\cos\theta + r_0^2}.$$

For $a/r_0 \ll 1$ (we recall that by the construction $a/r_0 \le 1/2$) this function has the value $f\left(\frac{a}{r_0}\right) \approx a^2/r_0^2$. Thus, for the contribution of wormholes we find

$$\delta\Lambda = \frac{1}{2}\int n(a, r_0)\left(1 - f\left(\frac{a}{r_0}\right)\right) da dr_0 = \frac{n}{2}\left(1 - \langle f\rangle\right).$$

From the above expression we see that to get the finite value of the cosmological constant $\Lambda_0 = \Lambda - \delta\Lambda < \infty$ one should consider the limit $n \to \infty$ (infinite density of virtual wormholes) which requires considering the smaller and smaller wormholes. From the other hand we have the obvious restriction

$\int 2n\frac{\pi^2}{2}a^4 dadr_0 < 1$, where $\frac{\pi^2}{2}a^4$ is the volume of one throat (wormholes cannot cut more, than the volume of space). Therefore the infinite density of wormholes remains consistent with a finite renormalization (38) (e.g., for the density of wormholes $n(a)$ it is sufficient to have the behavior $n(a) \sim 1/a$ as $a \to 0$). We also point out that the problem to determine the exact value for the cosmological constant Λ_0 requires to involve the simultaneous renormalization of all other fundamental constants (rest masses, charge values, and the gravitational constant).

7 Renormalization of charge values

The absolute value of the renormalized cosmological constant (e.g., see (29)) does not give the observed value yet. Indeed the behavior of the mean cutoff function at the mass-shall $\overline{N}(k)$ (38) defines some renormalization (screening) of the gravitational constant (of charge values) as well, while $\langle T_{\alpha\beta}(x)\rangle$ represents the source in (1). This leads to a somewhat reduced value of the cosmological constant. To find the observed value we have to account for two points. First is that the gas of virtual wormholes is actually dense ($n \to \infty$). And the second is that the polarization at wormholes has some part which leads to anti-screening (for the sake of simplicity we have neglected this part in (31) e.g., see for detail Ref.[8]).

7.1 dense gas

The expression (38) was obtained in the approximation of a rarefied gas. Indeed, only in this case the total bias obeys the superposition property (33). The dense gas can be accounted for as follows. Consider the equivalent description which is the introduction of the topological permeability $\widehat{\varepsilon}$, i.e. the modification of the equation in the form $\triangle\widehat{\varepsilon}G(x,x') = -4\pi^2\delta(x-x')$. In the case of the homogeneous and isotropic gas the relation between the bias b and $\widehat{\varepsilon}$ is simple in the Fourier representation, i.e. $\varepsilon(k) = 1/\left(1+b\left(k\right)\right) \approx 1-b\left(k\right)$. In the situation when $\widehat{\varepsilon} = const$ (e.g., at very large scales $k \to 0$) the topological permeability renormalizes merely the value of a source $\triangle G(x,x') = (4\pi^2/\widehat{\varepsilon})\delta(x-x')$ (or equivalently the value of the interaction constant $\gamma \to \gamma/\widehat{\varepsilon}$). Then (for a constant $\widehat{\varepsilon}$) the permeability of a dense gas can be obtained in the standard way. Indeed, let present $\varepsilon = 1+4\pi^2\chi$, where χ is the topological susceptibility of space. In linear approximation we get

$$4\pi^2\chi_0 \approx \int \pi^2 a^4 n\left(a,r_0\right)\frac{r_0^2}{a^2}dadr_0,$$

see (38). Then for a dense gas it is related to the linear susceptibility χ_0 as $\chi = \chi_0/\left(1-\pi^2\chi_0\right)$, e.g., see part 4.6 in Ref. [13]. Thus finally, we find the renormalization of the physical value of the interaction constant in the form

$$\gamma_{phys} = \frac{\gamma}{\widehat{\varepsilon}} = \gamma\frac{\left(1-\pi^2\chi_0\right)}{1+3\pi^2\chi_0} < \gamma.$$

7.2 renormalization of volume

The anti-screening part of the bias somewhat diminishes the reduction of the cosmological constant. It can be accounted for by the renormalization of the physical volume [8]. Indeed, wormholes merely cut some portion of the coordinate volume which is given by

$$V_{phys} = V\left(1 - \int \pi^2 a^4 n\,(a, r_0)\,dadr_0\right)$$

where the second term describes the volume of interiors of wormholes. Suppose that we work in the standard Euclidean space R^4 (we do not aware that the actual space has a smaller volume). Then we should introduce the apparent (or observed) interaction constant as

$$\gamma_{apparent} = \gamma_{phys}\frac{V}{V_{phys}} < \gamma$$

which accounts for the fact that the number density of strength lines remains the same, while the total number of strength lines fictitiously increases when we continue fields to the non-physical regions[5]. Thus for the observed value of the cosmological constant we get

$$\Lambda_{obs} = \gamma_{phys}\frac{V}{V_{phys}}\Lambda \ll \Lambda \sim 1.$$

8 Dark energy from actual wormholes

Consider now the contribution to the dark energy from the gas of actual wormholes. Unlike the virtual wormholes, actual wormholes do exist at all times and, therefore, a single wormhole can be viewed as a couple of conjugated cylinders $T^3_\pm = S^2_\pm \times R^1$. So that the number of parameters of an actual wormhole is less $\eta = (a, r_+, r_-)$, where a is the radius of $S^2_\pm$ and $r_\pm \in R^3$ is a spatial part of $R_\pm$.

For rigorous evaluation of dark energy in this case we, first, have to find the bias $b_1\,(x, x', \eta)$ analogous to (31) for the topology $R^4/(T^3_+ \cup T^3_-)$. There are many papers treating different wormholes in this respect (e.g., see Ref. [15] and references therein). However, in the present paper for an estimation we shall use a more simple trick.

[5]Such an effect can be easily analyzed on the simplest example of a cylinder $R^2 \times S^1$. In terms of R^3 the number density of degrees of freedom for the cylinder is always $N\,(k) < 1$. Indeed, consider the finite volume $V \in R^3$ and plane waves $f_k = \frac{1}{\sqrt{V}}e^{-ikx}$ which means that $k = 2\pi n/L$ $(V = L^3)$. Then $\sum_k = \int \frac{V}{(2\pi)^3} d^3k = \int \frac{V}{(2\pi)^3} N\,(k)\,d^3k$, i.e., for R^3 we have $N = 1$. In the case of the cylinder we find $\sum_k = \int \frac{L^2\ell}{(2\pi)^3} d^3k$, for small distances $k \gg k^* = 1/\ell$ (ℓ is the radius of the cylinder) and $\sum_k = \int \frac{L^2}{(2\pi)^2} d^2k$ as $k^* \gg k \gg k_0$, where $k_0 = 2\pi/L$. This defines $N_c\,(k) = V_{phys}/V = \ell/L < 1$ as $k \gg k^*$ and $N_c\,(k) = \pi/\,(Lk) < 1$ as $k^* \gg k \gg k_0$. Thus if we continue the cylinder to the whole space R^3 we set $\widetilde{N}_c = 1$ as $k \gg k^*$, while for scales $k^* \gg k$ we will get $\widetilde{N}_c = \pi/\,(\ell k) \gg 1$ which may be interpreted as the presence of dark matter (the scale dependent renormalization of the apparent mass $M_{apparent}\,(R) \sim M_0 R k^*$ [14]).

8.1 Beads of virtual wormholes

Indeed, instead of the cylinders $T^3_\pm$ we consider a couple of chains (beads of virtual wormholes $T^3_\pm \to \cup_n S^3_{\pm,n}$). Then the bias can be written straightforwardly

$$b_1\left(x,x',\eta\right)=\sum_{n=-\infty}^{+\infty}\frac{1}{2\pi^2 a}\left(\frac{1}{\left(R_{-,n}-x'\right)^2}-\frac{1}{\left(R_{+,n}-x'\right)^2}\right)\times \tag{39}$$

$$\times\left[\delta(\left|\vec{x}-\vec{R}_{+,n}\right|-a)-\delta(\left|\vec{x}-\vec{R}_{-,n}\right|-a)\right],$$

where $R_{\pm,n}=(t_n,r_\pm)$ with $t_n=t_0+2\ell n$ and $\ell\geq a$ is the step. We may expect that upon averaging over the position $t_0\in[-\ell,\ell]$ the bias for the beads will reproduce the bias for cylinders $T^3_\pm$ (at least it looks like a very good approximation). We point out that the averaging out $\frac{1}{2\ell}\int_{-\ell}^{\ell}dt_0$ and the sum $\sum_{n=-\infty}^{+\infty}$ reduces to a single integral $\frac{1}{2\ell}\int_{-\infty}^{\infty}dt$ of the zero term in (39). And moreover, the resulting total bias corresponds merely to a specific choice of the distribution function $F(\xi)$ in (33). Namely, we may take

$$NF(\xi)=\frac{1}{2\ell}\delta\left(t_+-t_-\right)f\left(\left|r_+-r_-\right|,a\right),$$

where $R_\pm=(t_\pm,r_\pm)$ and $f\left(s,a\right)$ is the distribution of cylinders, which can be taken as ($\widetilde{n}$ is 3-dimensional density)

$$f\left(\eta\right)=\frac{\widetilde{n}\left(a\right)}{4\pi r_0^2}\delta\left(s-r_0\right).$$

Using the normalization condition $\int NF(\xi)d\xi=N$ we find the relation $N=\frac{1}{2\ell}\widetilde{n}V=nV$, where n is a 4-dimensional density of wormholes and $1/\left(2\ell\right)$ is the effective number of wormholes on the unit length of the cylinder (i.e., the frequency with which the virtual wormhole appears at the positions $r_\pm$). This frequency is uniquely fixed by the requirement that the volume which cuts the bead is equal to that which cuts the cylinder $\frac{4}{3}\pi a^3=\frac{\pi^2}{2}a^4\frac{1}{2\ell}$ (i.e., $2\ell=\frac{3\pi}{8}a$ and $n=\frac{8}{3\pi a}\widetilde{n}$). Thus, we can use directly expression (35) and find (compare to (37)

$$b\left(k\right)=-\int 2n\left(a\right)a^2\frac{4\pi^2}{k^2}\left(1-\frac{\sin\left|\mathbf{k}\right|r_0}{\left|\mathbf{k}\right|r_0}\right)\frac{J_1\left(ka\right)}{ka/2}da, \tag{40}$$

where $k=(k_0,\mathbf{k})$. Here the first term merely coincides with that in (37) and, therefore, it gives the contribution to the cosmological constant $\Lambda=-n/2=-4\widetilde{n}/\left(3\pi a\right)$, while the second term describes a correction which does not reduce to the cosmological constant and requires a separate consideration.

8.2 Stress energy tensor

From (27) we find that the stress energy tensor

$$-\left\langle\Delta T_{\alpha\beta}\left(x\right)\right\rangle=\int\frac{k_\beta k_\alpha-\frac{1}{2}g_{\alpha\beta}k^2}{k^2}b\left(k,\xi\right)\frac{d^4k}{\left(2\pi\right)^4} \tag{41}$$

reduces to the two functions

$$T_{00} = \varepsilon = \lambda_1 - \frac{1}{2}\mu$$

$$T_{ij} = p\delta_{ij}, \quad p = \frac{1}{3}\lambda_2 - \frac{1}{2}\mu$$

where $\varepsilon + 3p = -\mu$ and $\lambda_1 + \lambda_2 = \mu$ and these functions are

$$\lambda_1 = -\int \frac{k_0^2}{k^2} b \frac{d^4k}{(2\pi)^4}, \quad \lambda_2 = -\int \frac{|\mathbf{k}|^2}{k^2} b \frac{d^4k}{(2\pi)^4}, \quad \mu \ = -\int b \frac{d^4k}{(2\pi)^4}.$$

By means of the use of the spherical coordinates $k_0^2/k^2 = \cos^2\theta$, $|\mathbf{k}|^2/k^2 = \sin^2\theta$, and $d^4k = 4\pi \sin^2\theta k^3 dk d\theta$ we get

$$\lambda_1 = \frac{n(a, r_0)}{2}\left(1 - 2\left(\frac{a}{r_0}\right)^2 f_1\left(\frac{a}{r_0}\right)\right), \quad \lambda 2 = \frac{3n(a, r_0)}{2}\left(1 - \frac{2}{3}\left(\frac{a}{r_0}\right)^2 f_2\left(\frac{a}{r_0}\right)\right)$$

where

$$f_{\binom{1}{2}}\left(\frac{a}{r_0}\right) = \frac{2}{\pi}\int_0^\pi \int_0^\infty \sin(x\sin\theta) \frac{J_1\left(\frac{a}{r_0}x\right)}{\frac{a}{r_0}x/2} \binom{\cos^2\theta}{\sin^2\theta} \sin\theta dx d\theta.$$

For $\frac{a}{r_0} \ll 1$ we find

$$f_1\left(\frac{a}{r_0}\right) \approx \left(1 + o\left(\frac{a}{r_0}\right)\right), \quad f_2\left(\frac{a}{r_0}\right) \approx \left(1 + o\left(\frac{a}{r_0}\right)\right).$$

Thus, finally we find

$$\varepsilon \simeq -\frac{n}{2} = -\frac{4\widetilde{n}}{3\pi a}, \quad p \simeq \varepsilon\left(1 - \frac{4}{3}\left(\frac{a}{r_0}\right)^2\right) = -\frac{4\widetilde{n}}{3\pi a}\left(1 - \frac{4}{3}\left(\frac{a}{r_0}\right)^2\right).$$

Which upon the continuation to the Minkowski space gives the equation of state in the form[6]

$$p = -\left(1 - \frac{4}{3}\left(\frac{a}{r_0}\right)^2\right)\varepsilon.$$

We point out that on the contrary to the virtual wormholes the energy density of actual wormholes is always finite $-\varepsilon < \infty$.

9 Summary

Thus, we see that virtual wormholes indeed lead to the regularization of all divergencies in QFT and form the local value of the cosmological constant.

[6]An arbitrary gas of wormholes splits in fractions with a fixed a and r_0.

This automatically distinguishes the Starobinsky model of inflation [6] as the model of the eternal Universe. It is impossible to say at which inflationary cycle we are.

In the case of actual wormholes the leading contribution to the stress energy tensor gives the negative cosmological constant. This happens due to the general fact that wormholes always remove some portion of field modes. Phenomenologically, such a removal can be prescribed to the presence of ghost fields. Virtual ghost particles reflect virtual wormholes, while real ghosts will describe wormholes created. On the inflationary stage production of ghost particles (of actual wormholes) essentially reduces the physical value of the cosmological constant which leads to a somewhat more rapid escape from the inflationary regime. We also point out that particle production during the inflation automatically generates huge rest masses of wormholes [3] which should lead to some complex picture for the escape from inflation. We also point out that the relating (see Sec.6) dynamic parameters of wormholes (or ghost fields) to the cutoff function requires further investigation.

We also point out that in the case of virtual wormholes the topological polarization effects (e.g., see the analogous consideration of actual wormholes in Ref. [3]) induce "dark charge" values on throats. This explains the well-known phenomenon of the dark charge (or the Higgs sector) in particle physics. We however consider this problem elsewhere.

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